MATH 135 Lecture Notes: Section 3.1 Limits
September 12, 2006

Continuity (informal definition): For a function to be continuous at a point, the function must be defined at the point and any small change in *x* produces only a small change in *f(x)*. (page 70)

Notice how this definition is applied in Examples 1-5, pages 70-71.

Notation: $x \to 2$ read "x approaches 2." When we say $x \to 2$ we mean that *x* may take on any value as close to 2 as desired, but it also means that *x* cannot be set equal to 2. (page 71)

Example 6 (page 71) shows that as $x \to 2$, $f(x) = 2x + 1 \to 5$.

Continuity (more formal definition): A function *f(x)* is continuous at $x = a$ if all three of these conditions are satisfied.

1. *f(a)* exists. 2. $\lim_{x \to a} f(x)$ exists. 3. $\lim_{x \to a} f(x) = f(a)$. (page 72)

Example 6 (page 71) shows how a table may be used to evaluate a limit. This may also be done with a calculator. Let Y1 = 2X + 1. Press 2ndTblSet and select Ask for the independent variable and Auto for the dependent variable. Then press 2ndTABLE and enter values for *x*: 3, 2,5, 2.1, 2.01, 2.001 and 1, 1.5, 1.9, 1.99, 1.999. Observe that as *x* approaches 2 from the right and from the left, *f(x)* approaches 5.

Examples 7 and 8 (pages 72 and 73) show that a limit may exist at a point of discontinuity. We may again explore the concept with a table.

Let Y2 = (2X^2 – 3X – 2)/(X – 2) and proceed as above.

Notice the difference in the graphs on page 73. Figure 3-8(a) shows a "punctured" line with a discontinuity ("hole") at (2,5), corresponding with the rational function. Figure 3-8(b) shows a solid line, corresponding with the function $y = f(x) = 2x + 1$.

Shortcuts for finding limits are shown in Examples 10 and 11 (page 74). In Example 10 we use the fact that the rational expression is equivalent to $x + 2$ for all values of *x* except for $x = 2$.

Skip Example 1 on page 74 and the paragraph which follows.

Limits with Infinity

The symbol ∞ is read as "infinity," **but must not be thought of as a number.** It is simply a symbol that stands for a <u>process</u> of considering numbers that become larger without bound (page 53).

Example 9 (page 73) shows what happens with a vertical asymptote.
As *x* approaches 2 from the right, $f(x) \to +\infty$.
As *x* approaches 2 from the left, $f(x) \to -\infty$.
Thus we say that $\lim_{x\to 2} f(x)$ does not exist.

Examples 13 and 14 (page 75) show what happens with a horizontal asymptone.
As $x \to \infty$, *f(x)* approaches a particular value.

In Example 14 we may also show that as $x \to -\infty$, $f(x) \to 0.5$.

Exercises: Pages 76-77

1, 3, 7-12, 17, 19, 21, 31, 41, 45, 47, 55